5.6.5 Exercice 5

Pour le système de l'exercice 3 on donne les déplacements initiaux de $x_1(0) = \delta$ et $x_2(0) = -\delta$ et est libéré du repos. Déterminez la réponse résultante du système.

Correction de l'exercice 5

$$\begin{cases} x_1(t) = A_{11}\sin(\omega_{01}t + \phi_1) + A_{12}\sin(\omega_{02}t + \phi_2) \\ x_2(t) = A_{21}\sin(\omega_{01}t + \phi_1) + A_{22}\sin(\omega_{02}t + \phi_2) \end{cases}$$

$$\begin{cases} x_1(t) = A_{11}\sin(\omega_{01}t + \phi_1) + A_{12}\sin(\omega_{02}t + \phi_2) \\ x_2(t) = 1.85A_{11}\sin(\omega_{01}t + \phi_1) - 0.161\ A_{12}\sin(\omega_{02}t + \phi_2) \end{cases}$$

$$x(t) = A_{11}\begin{pmatrix} 1 \\ 1.85 \end{pmatrix}\sin\left(0.391\sqrt{\frac{k}{m}}t + \phi_1\right) + A_{12}\begin{pmatrix} 1 \\ -0.16 \end{pmatrix}\sin\left(1.47\sqrt{\frac{k}{m}}t + \phi_2\right)$$

En appliquant les conditions initiales on obtient :

$$\begin{cases} x_1(0) = A_{11}\sin(\phi_1) + A_{12}\sin(\phi_2) = \delta \\ x_2(0) = 1.85A_{11}\sin(\phi_1) - 0.161\ A_{12}\sin(\phi_2) = -\delta \end{cases}$$

$$\begin{cases} \dot{x}_1(t) = 0.391A_{11}\cos(\phi_1) + 1.47A_{12}\cos(\phi_2) = 0 \\ \dot{x}_2(t) = (0.391)\,1.85A_{11}\sin(\phi_1) - (1.47)\,0.161\ A_{12}\sin(\phi_2) = 0 \end{cases}$$

$$\Rightarrow \begin{cases} \cos\phi_1 = \cos\phi_2 = \frac{\pi}{2} \\ A_{11} = -0.403\,\delta\ et\ A_{21} = 1.403\,\delta \end{cases}$$

$$x(t) = \delta\begin{pmatrix} 0.403 \\ 0.746 \end{pmatrix}\sin\left(0.391\sqrt{\frac{k}{m}}t + \frac{\pi}{2}\right) + \delta\begin{pmatrix} 1.403 \\ -0.254 \end{pmatrix}\sin\left(1.47\sqrt{\frac{k}{m}}t + \frac{\pi}{2}\right)$$